Mobile Robot Navigation Using a Sensor-Based Control Strategy

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Abstract

This work presents a new methodology for navigating mobile robots in unknown environments based on the robot's perception. The interaction between the robot and the environment is modelled and control laws are designed so that the robot is constrained to move on the Voronoi Diagram of the environment. The proposed method enables the robot to explore an unknown scene without any reference trajectory or any prior knowledge about the environment and avoiding the obstacles. We are applying this approach on an indoor mobile robot using a 2-D Laser Range Finder. We show that the displacement errors remain bounded when the movements of the robot are constrained by sensor based control laws, that results in a precise self-localization and reliable map building. Experimental results shown in this article validate this methodology.

1 Introduction

A safety navigation in an unknown environment requires some essential capabilities such as perception of the environment, reliable localization and maintaining an accurate representation of its workspace. Classically, path planning methods are stated in terms of finding a free trajectory given a priori known map of the environment without any perception by the robot [8]. In a second level researches have been done in the localization of the robot with simultaneous map building. Then, the robot is equipped with sensors and the perception data are used both in the localization and in the map building aspects. However very often a reference trajectory in the robot workspace is required [6]. A third level of research consider the perception capabilities of the robot to execute a reactive navigation in the workspace but without a

reliable localization nor a sufficient description of the environment [3].

We propose in this work a more complete approach that considers the perception of the environment embedded in a Sensor-Based Control strategy and, consequently, notably enhances the localization and simultaneous map building process. The basic functionalities are shown in figure (1). We use a 2-D
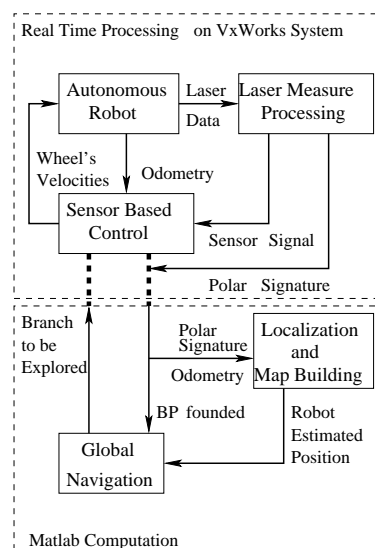
Figure 1: *The real time and off-line computation scheme.*

Laser Range Finder mounted on a mobile platform that delivers a planar cross section of the visible environment; this range information is processed and used in a closed loop control scheme to constrain the robot to reach and to move on the *Generalized Voronoi Diagram* of the environment, [2]. The *Generalized Voronoi Diagram* is defined as a set of points equidistant to two objects O_i and O_j, such that each point in this set is closer to O_i and O_j than any other objects O_k in the environment, $k \neq i, j$. The VD structure satisfies the following properties:

- to be locally defined for each current location.

- to belong to the free space.
- to ensure a complete exploration of the environment.
- to capture topology and accessibility of the map for a robot with a given size.

which are essential for the success of our methodology. The VD is particularly interesting due to the fact that it can be incrementally built from the laser data in a straightforward manner.

It is shown in figure (2) an example of a Voronoi Diagram (VD) of a bounded environment with a rectangular obstacle. When the robot arrives in a neigh-
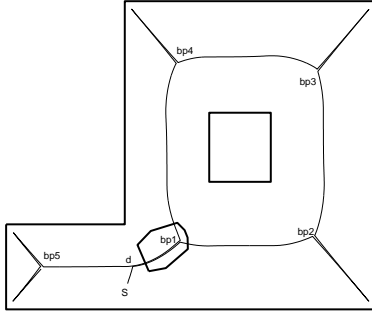

Figure 2: *The VD is formed by Edges connected by Bifurcation Points (pb$_i$). The robot starts at point S, reaches the VD at d, goes to the pb$_1$ and navigates the free workspace.*

bourhood of a Bifurcation Point (BP), figure (2), it is stabilised to this point, and a message is sent from the real time system to the hight level controllers (dashed line in figure (1)) and the localization and the incrementally map building are performed. Then the Global Navigation Manager will define a suitable branch to be explored based on the possible directions associated to the BP found and on the estimated position of the robot. Using a sensor-based control technique with the laser data as feedback we ensure a bounded error during the robot displacement which guarantees the unicity of the VD constructed and more reliable localization of the robot. In this paper, we focus on the sensor based control aspects, the localization and map building aspects can be found in [7].

The paper is organized as follows. In the section 2 we present the processing of the laser measurements using a Hough transform. The section 3 is devoted to the sensor based control aspects. Results obtained in our experimental testbed are also discussed. Finally, we conclude with some comments and open issues in section 4 .

2 Laser Measurement Processing

The perception system is formed by a laser range finder with a scanning device mounted on the mobile robot. The initial data delivered by the laser is a horizontal cross section of the robot's environment which is used in the localization and map building algorithm and as input to the sensor-based control as shown in this section. The range data, figure (3), is represented in a polar form:
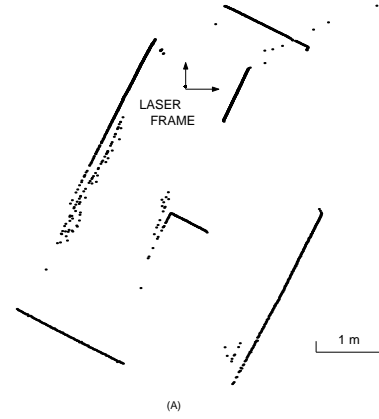

Figure 3: *Cross section of the environment represented in the laser frame.*

$$I = \{\delta(\theta_0), \ldots, \delta(\theta_i), \ldots, \delta(\theta_{2\pi})\} \qquad (1)$$

where $\delta_i = \delta(\theta_i)$ is the distance from the origin of the laser frame to the closest object at the angular position θ_i. Then we extract from (1) the set of the minimum distances between the laser and the polygonal visible objects:

$$I_{min} = \{(\delta, \theta)_0, \ldots, (\delta, \theta)_i, \ldots, (\delta, \theta)_n\} \qquad (2)$$

The set I_{min} corresponds to the parameters of the support lines associated to the polygonal objects in the environment and it can be obtained by the application of a parameter identification method respecting the processing time requirements. The Hough transform gives us the parameters of all visible objects and it is robust face to the noisy data, figure (3), that is corrupted by outliers points and reflectance effects.

The polar signature is processed using a Hough transform algorithm. The maximums of the accumulator space associated to a point (δ, θ) in the parameter space is then computed as shown in figure (4). These points constitutes the set I_{min}, (2). The signal feedback used in the control will be the two minimum distances (or three at a BP) in the set I_{min}.
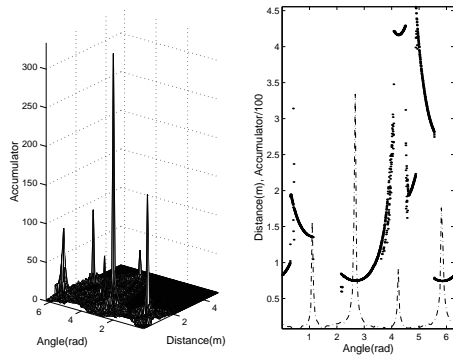
Figure 4: *(Left) Each peak corresponds a line with parameters (ρ, θ); (Right) the dashed curve represents the maxima for each direction in the parameter space. When there are more than one peak in a given direction (example two parallel support lines, peaks around directions 4rad or 6rad in the left) the nearest object to the robot is selected.*

3 Sensor-Based Control

Sensor-based Control and Task Function framework have been initially established in Samson et al. [1]. These methodologies have been extended to the case of visual servoing in Espiau et al. [5]. Some vision based tasks are shown in Chaumette and Rives [4] where low-level primitives, like points, lines, and spheres are considered as features used in the control loop.

3.1 Modelling

Let us assume a sensor rigidly mounted on the robot that delivers a signal s only dependent on the relative pose $\overline{r}$ between the sensor frame and the reference frame attached to the environment ($s: \mathcal{SE}_3 \to \mathbb{R}$, where $\mathcal{SE}_3$ represents the Special Euclidean group and $\overline{r}$ is an element of $\mathcal{SE}_3$). The derivative of the signal s can be written as:

$$\dot{s} = \frac{\partial s}{\partial \overline{r}} \cdot \frac{\partial \overline{r}}{\partial t} = L^T T_{SE} \tag{3}$$

where $T_{SE} = (V, \omega)$ is the velocity screw between the sensor and the perceived environment; and L^T is a matrix that represents the interaction between the sensor and the robot's environment called *Interaction matrix* (for generalities and more details about sensor-based control and task function approach see [1]).

The interaction with the environment, in our case, is done by a 2-D laser range scanner which provides a planar cross section of the environment, $\{\delta(\theta_0), \ldots, \delta(\theta_i), \ldots, \delta(\theta_{2\pi})\}$; let us note $\delta_i = \delta(\theta_i)$ the distance from the origin of the laser frame (L) to the nearest object O_i at the angular position θ_i, so we can
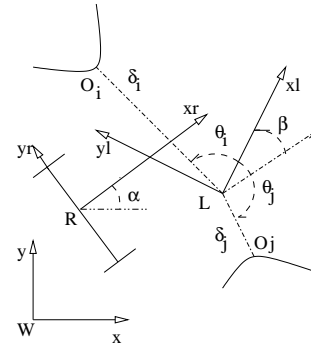

Figure 5: *The unicycle robot R carries a 2-D laser with frame in L, it's shown in the figure the distances $\delta(\theta_i)$, $\delta(\theta_j)$ from the laser to the objects O_i and O_j, the robot's orientation in the world frame W is α and the laser's orientation in W is $\beta + \alpha$.*

choose as sensor signal $s_i = \delta_i$, figure (5). Let us consider that the laser frame L is moving with a velocity screw $\vec{\tau}_l = (\vec{V}_l, \vec{\Omega}_l)$ where $\vec{V}_l$ and $\vec{\Omega}_l$ are respectively the translational and rotational velocities expressed in the fixed reference frame. Then, the variation of the sensor signal can be expressed, after some simple kinematic calculation (see [1] for details) :

$$\dot{s}_i = -\frac{1}{< \vec{n_O} \cdot \vec{n_{\theta_i}} >}[< \vec{n_O} \cdot \vec{V}_l > +$$
$$\delta_i < (\vec{n_O} \times \vec{n_{\theta_i}}) \cdot \Omega_l >] \tag{4}$$

where $< ., . >$ is the usual scalar product. $\vec{n_{\theta_i}}$ is the unit vector in the direction of measurement and $\vec{n_O}$ is the unit vector normal to the surface of the object at the impact point of the laser beam. The Eq. (4) constitutes the model of the interaction between the laser's frame and the environment. Considering the couple (δ_i, θ_i) in figure (5), such that δ_i is orthogonal to the surface of the object O_i so that $\vec{n_O} = -\vec{n_{\theta_i}} = [-\cos(\theta_i) -\sin(\theta_i)]$. The variation $\dot{\delta}_i$ is obtained from the interaction model in Eq.(4):

$$\dot{\delta}_i = \begin{bmatrix} -\cos\theta_i & -\sin\theta_i & 0 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \Omega \end{bmatrix} \tag{5}$$

3.2 Robot Task Definition

Using the *task function* framework, we want to constrain the laser frame to move on the Voronoi diagram and to reach the nearest BP. These behaviour are modelled as a *task function* e, function of the sensor signals (δ_i, θ_i), such that if $e = 0$ the tasks are perfectly achieved. Such tasks e_1, e_2 and e_3 are defined below.

3.2.1 Moving along the Voronoi Diagram

Considering the set I_{min}, we can state that :

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- The origin of the laser frame is on a branch of the VD if, and only if, it exists two couples (δ_i, θ_i) and (δ_j, θ_j) so that $\delta_i = \delta_j = \min \delta \{I_{min}\}$.
- The x-axis of the laser frame is colinear to the VD direction if, and only if, $\theta_i = -\theta_j$.

A task function e_1 such that $e_1 = 0$ satisfies these two statements can be defined such that :

$$e_1(r) = \begin{pmatrix} \delta_i - \delta_j \\ \frac{\theta_i + \theta_j}{2} \end{pmatrix} \quad (6)$$

and using Eq.(5) yields :

$$\dot{e}_1 = L^T \begin{bmatrix} V_x \\ V_y \\ \Omega \end{bmatrix} \quad (7)$$

where :

$$L^T = \begin{bmatrix} (c(\theta_2) - c(\theta_1)) & (s(\theta_2) - s(\theta_1)) & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (8)$$

with $c(\theta_i) = \cos(\theta_i)$ and $s(\theta_i) = \sin(\theta_i)$.

When the regulation to zero of e_1 is perfectly achieved then, the origin of the laser frame belongs on the Voronoi Diagram and its x-axis is tangent to it.

Let us now analyse the dimension of the space spanned by the interaction matrix (8). $Dim(L^T) = 2$ that means that only 2 degrees of freedom are constrained by the task e_1. The motion which remains free belongs to the null space of L^T such that :

$$\tau^\star = Ker(L^T) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad (9)$$

this motion, along the x-axis, which does not perturbate the main task e_1 can be used, in a secondary task, to control the robot's motion along the Voronoi Diagram.

The secondary task function can be specified as a minimization of a cost function e_2, with gradient $g_s = \frac{\partial e_2}{\partial r}$ evaluated under the constraint of the perfect regulation of e_1. A general task function e_{g1} (*Hybrid Task*) can be then designed using the redundant task framework described in [1] :

$$e_{g1} = W^+ e_p + \alpha_t (I_n - W^+ W) g_s^T \quad (10)$$

- α_t is a positive weight tuning the principal and the secondary tasks, $(\alpha_t \leq 1)$.
- W is a $m \times n$ full-rank matrix, so that $Ker(W) = Ker(L^T)$, m is the dimension of e_1 and n the number of degrees of freedom of the laser frame.
- $(I_n - W^+ W)$ is the orthogonal projection operator onto the null space of W, I_n represents the $n \times n$

identity matrix.
- W^+ pseudo-inverse of W.
Introducing $e_p = Ce_1$, where C is a combination matrix, so that with $CL^T W > 0$ the regularity of e is guaranteed. A good choice is $C = WL^{T^+}$, [1].

3.2.2 Stabilizing on a Bifurcation Point

The stabilization task to the BP, called e_3 is then considered. It takes place when the robot is in a neighbourhood of a BP and it is defined as:

$$e_3(r) = \begin{pmatrix} \delta_1 - \delta_2 \\ \delta_1 - \delta_3 \\ \frac{\theta_1 + \theta_2}{2} \end{pmatrix} \quad (11)$$

where (δ_1, θ_1), (δ_2, θ_2) and (δ_3, θ_3) are the distances with its measurement's orientations from the laser frame to the three near objects. Using eq.(5) yields :

$$\dot{e}_3 = L^T \begin{bmatrix} V_x \\ V_y \\ \Omega \end{bmatrix} \quad (12)$$

where :

$$L^T = \begin{bmatrix} (c(\theta_2) - c(\theta_1)) & (s(\theta_2) - s(\theta_1)) & 0 \\ (c(\theta_3) - c(\theta_1)) & (s(\theta_3) - s(\theta_1)) & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (13)$$

with $c(\theta_i) = \cos(\theta_i)$ and $s(\theta_i) = \sin(\theta_i)$.
Analysing the dimension of the space spanned by the interaction matrix (13) we see that,

$$Ker(L^T) = \{\emptyset\} \quad (14)$$

and L^T is a full rank interaction matrix. There is no secondary task compatible with this stabilizing task, $g_s^T = 0$, and from (10) we have :

$$e_{g2} = W^+ Ce_3 \quad (15)$$

- W is chosen $W = I_3$, so that $Ker(W) = Ker(L^T)$.
- $C = WL^{T^{-1}}$ is the combination matrix.
Then,

$$e_{g2} = L^{T^{-1}} e_3 \quad (16)$$

When the stabilization on a BP is finished the new branch information is sent to the robot with the new direction to be explored and its two associated distances. The robot starts exploring this new direction with the task (6), the third distance of the local BP is monitored so that a backward stabilization is avoided as the robot goes away toward a new BP.

3.3 The Control Law

We are interested in a velocity control law which ensures an exponential decay $\dot{e} = -\lambda e$ of the task

functions (10) and (16) (λ determines the convergence rate). Computing the derivative of e_{g1} which is r and time dependent, yields :

$$\dot{e}_{g1} = -\frac{\partial e_{g1}}{\partial r}\tau_l + \frac{\partial e_{g1}}{\partial t} \qquad (17)$$

and the desired velocity screw of the laser frame is given by,

$$\tau_l = \left(\frac{\partial e_{g1}}{\partial r}\right)^{-1}(-\lambda e_{g1} - \frac{\partial e_{g1}}{\partial t}) \qquad (18)$$

where $\frac{\partial e_{g1}}{\partial t}$ is calculated considering (10),

$$\frac{\partial e_{g1}}{\partial t} = W^+\frac{\partial e_p}{\partial t} + \alpha_t(I_n - W^+W)\frac{\partial g_s}{\partial t} \qquad (19)$$

($\frac{\partial e_p}{\partial t}$ is introduced to account an eventual motion of the object's frame which in our case is static, so $\frac{\partial e_p}{\partial t} = 0$). We can consider $\frac{\partial e_{g1}}{\partial r} = I_3$ for control purposes and the desired velocity control input τ_l, will be :

$$\tau_l = -\lambda e_{g1} - \alpha_t(I_n - W^+W)\frac{\partial g_s}{\partial t} \qquad (20)$$

The secondary task is then considered as a motion on the VD with a desired velocity $V_d = [v_{dx}v_{dy}]^T$ in the world frame. We consider a constraint $F = X(t) - X_0 - V_d.t = 0$, where X is the position of the laser in the world frame. The cost function to be minimized is $e_2 = \frac{1}{2}F^2$ and its gradient $g_s = \frac{\partial e_2}{\partial r}$ expressed in the laser frame is done by:

$$g_s = R_{WL}^T F \qquad (21)$$

where R_{WL} is the rotation matrix between the laser frame L and the world frame W, figure (5). The velocities v_{d_x} and v_{d_y} are corrected by a first-order linear filter for stability purposes as follows:

$$V_d = \alpha_1(X(t) - X_d(t)) + V_{d1} \qquad (22)$$

and $X_d(k+1) = (1 - \beta_1)X_d(k) + \beta_1 X(k) + V_d(k).\Delta t$ in a numerical representation. $\alpha_1 > 0$ and $\beta_1 > 0$ are tuning parameters. The hybrid task (10) is calculated with the interaction matrix L^T as in (8), and W chosen as,

$$W = \begin{bmatrix} 0 & I_2 \end{bmatrix} \qquad (23)$$

The control of the robot that carries the sensor is then calculated. Let us consider the figure (5), the controlled frame is fixed at the point L of the robot. The state of the robot is $\{x, y, \alpha, \beta\}$. The fourth degree of freedown β was considered to circumvent the nonholonomic constraint of the mobile robot, it is

a virtual rotation movement of the controlled frame (Laser). The desired control input is calculated from (20), where $\theta_l = \beta + \alpha$ as in figure (5):

$$\tau_l = -\lambda e_{g1} + \alpha_t \begin{bmatrix} v_{dx}cos(\theta_l) + v_{dy}sin(\theta_l) \\ 0 \\ 0 \end{bmatrix} \qquad (24)$$

and for the stabilization task, 16, where $\frac{\partial g_s}{\partial t} = 0$, we have :

$$\tau_l = -\lambda e_{g2} \qquad (25)$$

Then the control of the robot is calculated as follow,

$$u = \begin{pmatrix} v \\ \omega \\ \dot{\beta} \end{pmatrix} = J_R^{-1}\tau_l \qquad (26)$$

and $v = \dot{x}cos(\alpha) + \dot{y}sin(\alpha)$ is the heading speed, ω is the angular velocity of the unicycle and J_R is the Jacobian matrix between the laser frame L and the robot frame R. It is shown in figures (6 -7)
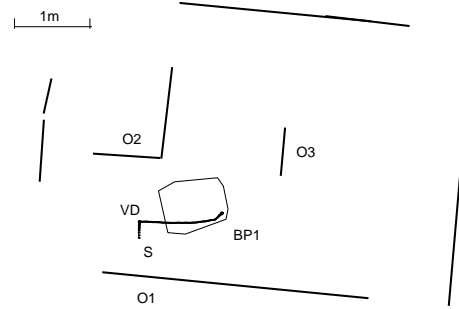

Figure 6: *The robot under the controls (24) and (25). The robot is stabilized to the vd and find the bp1. It's shown the global map in construction.*

the experimental result of the application of these control inputs. The robot is stabilized to the VD and displaces on it under the control (24), since the third object is detected the robot is then stabilized to the bifurcation point under the control (25).

This sensor-based control scheme has been validated with the Robot ANIS developed in the ICARE Robotic group at INRIA-Sophia Antipolis. This robot is equipped with a 2D-Laser Ranging Finder that gives 2000 points of measurements with a rate of 80ms, thanks to the microelectronics developed in our Laboratory.

We remind that the objective is to control the robot to reach the Voronoi diagram and then slide on it localizing itself and iteratively constructing the map of its environment. The experimental procedures
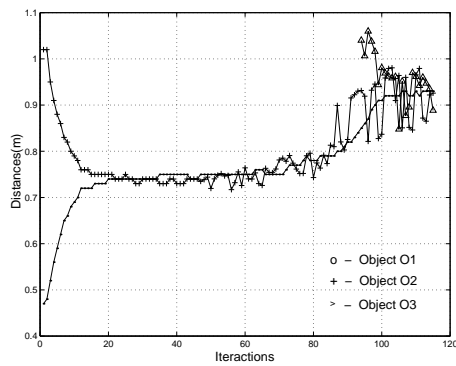
Figure 7: *The evolution of the laser's distances at the stabilization to the VD, displacement on the VD and stabilization to the BP. The objects O_1, O_2 and O_3 are shown in figure (6).*

are explained with the results shown in figure (8). When e_1, Eq.(6) in section 3, is realized the robot has reached the Voronoi diagram, point VD, then it moves on the branches connected by the bifurcation points, called BP; when the $BP's$ are reached the localization process takes place and the global map in construction is updated, then a suitable branch is chosen by the navigation strategy for continuing the exploration. We can see in fig.(8) the global map constructed when the robot arrives in the $BP5$, the trajectory realized by the laser frame is shown as well.
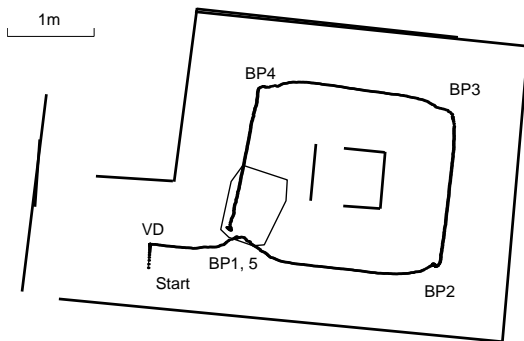

Figure 8: *The global map constructed.*

4   Conclusions

We have presented a sensor-based control strategy that enables the robot to navigating in an unknown environment, constructing the map and localizing itself, only based on the range sensor readings. The task functions that describe the interaction between the robot and the environment have been defined and experimentally applied. The robot can deal

with moving obstacles with the control methodology described in section 3, however the localization and map reconstruction algorithm is relevant in static environments only.

The next step in our researches consists in applying the method described above in natural indoor environments (corridors, rooms, etc.). An efficient perception processing taking in account the uncertainties of the data is important to achieve this objective. We are developing an approach that adds a probabilistic reasoning to the processing of the sensor data, based on the Bayesian inference techniques.

5   References

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